

# Resummation of High Order Corrections in Higgs Boson Plus Jet Production at the LHC

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## Abstract

We study the effect of multiple parton radiation to Higgs boson plus jet production at the LHC, by applying the transverse momentum dependent (TMD) factorization formalism to resum large logarithmic contributions to all orders in the expansion of the strong interaction coupling. We show that the appropriate resummation scale should be the jet transverse momentum, rather than the partonic center of mass energy which has been normally used in the TMD resummation formalism. Furthermore, the transverse momentum distribution of the Higgs boson, particularly near the lower cut-off applied on the jet transverse momentum, can only be reliably predicted by the resummation calculation which is free of the so-called Sudakov-shoulder singularity problem, present in fixed-order calculations.

*Introduction.* With the discovery of the Higgs boson at CERN LHC [1, 2], the high energy physics community is now focusing on determining the properties of the Higgs boson. This is done by carefully comparing the experimental measurements of total and differential cross sections in various Higgs boson production and decay channels to the Standard Model (SM) predictions [3]. Among them, both the ATLAS and CMS collaborations have reported results for several exclusive channels of Higgs production with zero, one or two jets [4–8]. With more data to be collected at the LHC, studying the Higgs boson plus multijet processes will allow us to further test the dynamics of perturbative QCD on the Higgs boson production, and to better discriminate various Higgs boson production mechanisms [3].

One particular example is the inclusive production of the Higgs boson plus one jet,

$$A(P) + B(\bar{P}) \rightarrow H(P_H) + Jet(P_J) + X, \quad (1)$$

where  $P$  and  $\bar{P}$  represent the incoming hadrons' momenta,  $P_H$  and  $P_J$  for final state particle momenta. With higher luminosity at Run II of the LHC, the experimental uncertainties of the cross section measurements of this process will be greatly reduced. Therefore, a precise theoretical evaluation will be required to test the production mechanism for the Higgs boson. The major theoretical uncertainty comes from higher order QCD corrections. In order to reduce this uncertainty, two methods can be applied: One is to compute the higher-order corrections in the expansion of the strong coupling constant  $\alpha_s$ ; another is to resum the large logarithms associated with the perturbative calculations to all orders in  $\alpha_s$ . Great progress has been made recently in fixed order computations with a next-to-next-to-leading order (NNLO) calculation completed for Higgs plus one jet production [9–12]. Meanwhile, the transverse momentum dependent (TMD) resummation [13–15] has been derived in Ref. [16] for this process, where the Sudakov double logarithms at low imbalance transverse momentum of the Higgs boson and the jet have been resummed to all orders in  $\alpha_s$ . While the fixed order calculation provides a better determination of the total production rate of the Higgs plus one jet events, it fails to predict the differential distribution of the Higgs boson transverse momentum, near the lower cut-off applied on the jet transverse momentum, which is known as the Sudakov-shoulder singularity problem [17]. Fortunately, this problem can be resolved by performing an all-order resummation calculation, as to be shown below.

The rest of this paper is organized as follows. We first introduce the TMD formalism for Higgs boson plus jet production, and discuss the factorization property, with the special emphasis on the scale dependence in the TMD factorization calculations. Then, we will extend the resummation formalism derived in Ref. [16] to predict various inclusive observables in Higgs boson plus jet production in  $pp$  collisions, by integrating over the imbalance transverse momentum of the Higgs boson and the final state jet which is zero at the leading order. For the inclusive cross sections of the process of (1), we have to integrate over a wide range of rapidity, where we encounter two separate large momentum scales: the partonic center of mass energy squared ( $s$ ) and the jet transverse momentum squared ( $P_{J\perp}^2$ ). As discussed below, the TMD factorization formalism indicates that the appropriate choice for the renormalization scale in our resummation calculation should be set around  $P_{J\perp}^2$ , rather than  $s$ .

*TMD Resummation.* In our calculation, the effective Lagrangian in the heavy top quark mass limit is used to describe the coupling between Higgs boson and gluon,

$$\mathcal{L}_{eff} = -\frac{\alpha_s}{12\pi v} F_{\mu\nu}^a F^{a\mu\nu} H, \quad (2)$$

where  $v$  is the vacuum expectation value,  $H$  the Higgs field,  $F^{\mu\nu}$  the gluon field strength tensor, and  $a$  the color index. Our TMD resummation formula can be written as [16]:

$$\frac{d^5\sigma}{dy_H dy_J dP_{J\perp}^2 d^2\vec{q}_\perp} = \sum_{ab} \sigma_0 \left[ \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} W_{ab \rightarrow Hc}(x_1, x_2, b_\perp) + Y_{ab \rightarrow Hc} \right], \quad (3)$$

where  $y_H$  and  $y_J$  denote the rapidities of the Higgs boson and the jet, respectively,  $P_{J\perp}$  the jet transverse momentum, and  $\vec{q}_\perp = \vec{P}_{H\perp} + \vec{P}_{J\perp}$  the imbalance transverse momentum of the Higgs boson and the jet. The first term ( $W$ ) contains all order resummation effect and the second term ( $Y$ ) accounts for the difference between the fixed order result and the so-called asymptotic result which is given by expanding the resummation result to the same order in  $\alpha_s$  as the fixed order term.  $\sigma_0 = \frac{4}{9} \frac{4\alpha_s^3 \sqrt{2} G_F}{s^2 (4\pi)^3}$  denotes the normalization of the differential cross section, and  $x_1$  and  $x_2$  are the momentum fractions of the incoming hadrons carried by the partons, with  $x_{1,2} = \frac{\sqrt{m_H^2 + P_{H\perp}^2} e^{\pm y_H} + \sqrt{P_{J\perp}^2} e^{\pm y_J}}{\sqrt{S}}$ . From the derivation of Ref. [16], we can write the all order resummation result for  $W$  as

$$W_{gg \rightarrow Hg}(x_1, x_2, b) = H_{gg \rightarrow Hg}(s, \hat{\mu}) x_1 f_g(x_1, \mu = b_0/b_\perp) x_2 f_g(x_2, \mu = b_0/b_\perp) e^{-S_{\text{Sud}}(s, \hat{\mu}^2, b_\perp)} \quad (4)$$

where  $s = x_1 x_2 S$ , where  $S$  is the partonic center of mass energy squared,  $b_0 = 2e^{-\gamma_E}$  with  $\gamma_E$  being the Euler constant,  $\hat{\mu}$  is the renormalization scale to apply the TMD factorization in the resummation calculation.  $\hat{\mu}$  is also the scale to define the TMDs in the Collins 2011 scheme [15].  $f_{a,b}(x, \mu)$  are the parton distribution functions(PDFs) for the incoming partons  $a$  and  $b$ , and the  $\mu$  is the evolution scale of the PDFs. The renormalization scale has been set as  $\hat{\mu}^2 = s$  in Ref. [16] to simplify the final expression, which is also an appropriate choice for describing experimental observables in the central rapidity region. In this paper, we will keep the renormalization scale explicitly in the above equation to demonstrate how to choose an appropriate scale for numeric calculations.

The Sudakov form factor can be expressed as:

$$S_{\text{Sud}}(b) = \int_{b_0^2/b^2}^{\hat{\mu}^2} \frac{d\mu^2}{\mu^2} \left[ \ln \left( \frac{s}{\mu^2} \right) A + B + D \ln \frac{1}{R^2} \right], \quad (5)$$

where  $R$  denotes the jet size of the final state jet. The coefficients  $A$ ,  $B$  and  $D$  can be expanded perturbatively in  $\alpha_s$ . For  $gg \rightarrow Hg$  channel, at one-loop order, we have  $A = C_A \frac{\alpha_s}{\pi}$ ,  $B = -2C_A \beta_0 \frac{\alpha_s}{\pi}$ , and  $D = C_A \frac{\alpha_s}{2\pi}$ . For  $gq \rightarrow Hq$  channel, we have  $A = (C_F/2 + C_A/2) \frac{\alpha_s}{\pi}$ ,  $B = (-C_A \beta_0 - 3/4 C_F - (1/2) C_A \ln u/t + (1/2) C_F \ln u/t) \frac{\alpha_s}{\pi}$ , and  $D = C_F \frac{\alpha_s}{2\pi}$ . By applying the TMD factorization in Collins 2011 scheme, we obtain the hard factor  $H_{gg \rightarrow Hg}$  in Eq. (4), at the one-loop order, as

$$\begin{aligned} H_{gg \rightarrow Hg}^{(1)} &= H_{gg}^{(0)} \frac{\alpha_s C_A}{2\pi} \left[ \ln^2 \left( \frac{\hat{\mu}^2}{P_{J\perp}^2} \right) + 2\beta_0 \ln \frac{\hat{\mu}^2}{P_{J\perp}^2 R^2} + \ln \frac{1}{R^2} \ln \frac{\hat{\mu}^2}{P_{J\perp}^2} - 6\beta_0 \ln \frac{\hat{\mu}^2}{\tilde{\mu}^2} - 2 \ln \left( \frac{P_{J\perp}^2}{\hat{\mu}^2} \right) \ln \left( \frac{s}{\hat{\mu}^2} \right) \right. \\ &\quad - 2 \ln \frac{s}{-t} \ln \frac{s}{-u} + \ln^2 \left( \frac{\tilde{t}}{m_h^2} \right) - \ln^2 \left( \frac{\tilde{t}}{-t} \right) + \ln^2 \left( \frac{\tilde{u}}{m_h^2} \right) - \ln^2 \left( \frac{\tilde{u}}{-u} \right) \\ &\quad \left. + 2\text{Li}_2 \left( 1 - \frac{m_h^2}{s} \right) + 2\text{Li}_2 \left( \frac{t}{m_h^2} \right) + 2\text{Li}_2 \left( \frac{u}{m_h^2} \right) + \frac{67}{9} + \frac{\pi^2}{2} - \frac{23}{54} N_f \right] + \delta H^{(1)}, \end{aligned} \quad (6)$$

where  $H^{(0)} = \frac{C_A}{4(N_c^2 - 1)} (s^4 + t^4 + u^4 + m_h^8) / (stu)$ , and  $s$ ,  $t$  and  $u$  are the usual Mandelstam variables for the partonic  $2 \rightarrow 2$  process, and  $\delta H^{(1)}$  represents terms not proportional to  $H^{(0)}$

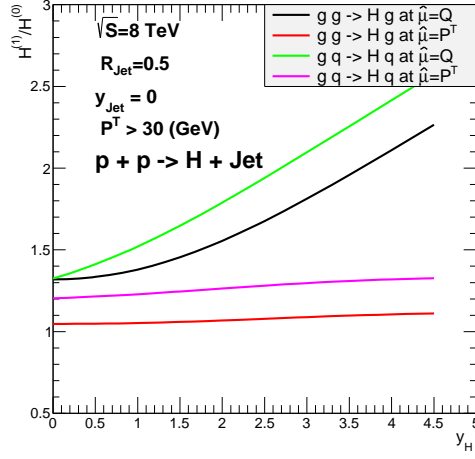


FIG. 1: The ratio of  $H^{(1)}/H^{(0)}$  as a function of Higgs boson rapidity.

and can be found in Refs. [18, 19]. We further introduce the shorthand notation  $\tilde{t} = m_h^2 - t$  and  $\tilde{u} = m_h^2 - u$ , and use  $\tilde{\mu}$  to denote the renormalization scale for  $\alpha_s$ . Similarly, for the subprocess  $g + q \rightarrow H + q$ , we have

$$\begin{aligned}
H_{gq \rightarrow Hq}^{(1)} = & H^{(0)} \frac{\alpha_s}{2\pi} \left\{ C_A \left[ \frac{1}{2} \ln^2 \left( \frac{\hat{\mu}^2}{P_{J\perp}^2} \right) + \ln \left( \frac{P_{J\perp}^2}{\hat{\mu}^2} \right) \ln \left( \frac{u}{t} \right) + \ln \left( \frac{P_{J\perp}^2}{\hat{\mu}^2} \right) \ln \left( \frac{s}{\hat{\mu}^2} \right) - 2 \ln \frac{-t}{\hat{\mu}^2} \ln \frac{-u}{\hat{\mu}^2} \right. \right. \\
& \left. \left. - 4\beta_0 \ln \frac{-u}{\hat{\mu}^2} - 6\beta_0 \ln \frac{\hat{\mu}^2}{\tilde{\mu}^2} + 2\text{Li}_2 \left( \frac{u}{m_h^2} \right) - \ln^2 \frac{\tilde{u}}{-u} + \ln^2 \frac{\tilde{u}}{m_h^2} + \frac{7}{3} + \frac{4\pi^2}{3} \right] \right. \\
& + C_F \left[ \frac{1}{2} \ln^2 \left( \frac{\hat{\mu}^2}{P_{J\perp}^2} \right) + \frac{3}{2} \ln \frac{\hat{\mu}^2}{P_{J\perp}^2 R^2} + \ln \frac{1}{R^2} \ln \frac{\hat{\mu}^2}{P_{J\perp}^2} - \ln \frac{P_{J\perp}^2}{\hat{\mu}^2} \ln \frac{u}{t} - \ln \frac{P_{J\perp}^2}{\hat{\mu}^2} \ln \frac{s}{\hat{\mu}^2} + 3 \ln \frac{-u}{\hat{\mu}^2} \right. \\
& \left. \left. + 2\text{Li}_2 \left( 1 - \frac{m_h^2}{s} \right) + 2\text{Li}_2 \left( \frac{t}{m_h^2} \right) - \ln^2 \left( \frac{\tilde{t}}{-t} \right) + \ln^2 \left( \frac{\tilde{t}}{m_h^2} \right) - \frac{3}{2} - \frac{5\pi^2}{6} \right] + 20\beta_0 \right\} + \delta H^{(1)}. \quad (7)
\end{aligned}$$

It is interesting to note that many of the logarithms in  $H^{(1)}$  can be eliminated if the factorization scale  $\hat{\mu}$  is chosen to be  $P_{J\perp}$ . To illustrate this point, we plot the ratio of  $H^{(1)}/H^{(0)}$  as functions of Higgs rapidity  $y_H$  in Fig. 1 with the jet rapidity fixed at  $y_j = 0$ . It shows that  $H^{(1)}$  is much larger than  $H^{(0)}$  in the large  $y_H$  region if we choose  $\hat{\mu}^2 = s$ . In contrast, the ratio of  $H^{(1)}/H^{(0)}$  becomes less sensitive to  $y_H$  with  $\hat{\mu}^2 = P_{J\perp}^2$ . This is because when the difference of  $y_H$  and  $y_J$  becomes large, the invariant mass  $Q^2$  of the Higgs boson and the leading jet can become much larger than the transverse momentum of the jet. Hence, we should choose  $\hat{\mu} = P_{J\perp}$  to resum the large logarithms in the perturbative contributions. In the following, we will adopt this scale choice in our theory predictions, though we will also show some results with  $\hat{\mu}^2 = s$  for the sake of comparison.

*Higgs Boson Plus Jet Production at the LHC.* We apply the above resummation formula to compute the differential and total cross sections of the Higgs boson production associated with a high energy jet. We will integrate out the imbalance transverse momentum  $q_\perp$  of Eq. (3), and take into account the contributions from both the  $gg \rightarrow Hg$  and  $gq(\bar{q}) \rightarrow Hq(\bar{q})$  channels. The  $gg$  and  $gq$  productions channel account for about 71% and 29% of the total rate, respectively. The  $q\bar{q} \rightarrow Hg$  channel contribution is less than about 1% and is ignored

in our calculations. We use the anti- $k_t$  algorithm to define the observed jet, and the jet size is set at  $R = 0.5$ . In our calculation we will apply the narrow jet approximation [20].

Before we present our numeric results, we would like to comment on the cross-check of this method. First, we perform the fixed order expansion of the integral of Eq. (3) to obtain the total cross section to compare with the fixed order prediction. The resummation formalism is designed in such a way that the  $Y$ -term vanishes as  $q_\perp$  approaches to zero. Hence, the contribution to the total cross section from the small  $q_\perp$  region mainly comes from the integration of the  $W$ -term from  $q_\perp = 0$  to a small value  $\Lambda$  (about 1 GeV). After expanding its result to the  $\alpha_s$  order and summing up with the fix-order contribution for  $q_\perp$  greater than  $\Lambda$ , we obtain the total cross section at the  $\alpha_s$  order [21]. We find that our result differs from the exact NLO result given by the MCFM code [22], which also uses heavy top quark effective theory in the calculation, by about 2% for  $R = 0.5$ , with the hard scale set to be the Higgs boson mass. This discrepancy arises from the narrow jet approximation made in calculating the  $H^{(1)}$  term, and it increases as  $R$  increases. We could model this difference as an additional  $R$ -dependent function inside  $H^{(1)}$ . Through comparison between our result in the small  $R$  limit and the full result from MCFM for different  $R$  values ranging from 0.3 to 0.7, and assuming the higher  $R$  correction is proportional to  $H^{(0)}$ , we parameterize the correction as  $H^{(0)} \frac{\alpha_s}{2\pi} (C_A R - 1.1 R + 23.3 R^2)$  and  $H^{(0)} \frac{\alpha_s}{2\pi} (C_F R - 0.8 R + 22.3 R^2)$  in  $H^{(1)}$  for producing the final state gluon and quark jets, respectively. These modifications will be included in the following numeric calculations. Second, the  $q_\perp$  in the resummation part is required to be smaller than the renormalization scale  $\hat{\mu}$  so that the Sudakov factor will go to one when  $q_\perp$  is integrated out. Namely, in our numerical calculations, we have included a theta-function  $\Theta(\hat{\mu} - q_\perp)$  in Eq.(3) to limit the range of  $q_\perp$  integration. This will result in a similar total cross section predicted from our resummation calculation as that from the fixed order calculation, though they differ in the differential distributions of some inclusive experimental observables, such as  $(P_{H\perp})$ . Furthermore, as shown in Ref. [23], the heavy top quark effective theory does not approximate well the exact one-loop calculation with full  $m_t$  dependence included, when  $P_{H\perp}$  is much larger than the top quark mass  $m_t$ .

In our numeric calculations, we have included in the Sudakov form factor, in addition to the  $A^{(1)}$ ,  $B^{(1)}$  and  $D^{(1)}$  contributions discussed above, the  $A^{(2)}$  contribution at the two-loop order. This is because the coefficient  $A^{(2)}$  only depends on the flavor of the incoming partons and is the same as that for inclusive Higgs boson production via  $gg \rightarrow H + X$  [24]. Following the experimental analysis [7], we require the rapidity of the observed jet to satisfy  $|y_J| < 4.4$ . Since our numerical results are obtained using the heavy top quark effective theory, the effect from finite quark mass in the loops is not included [22]. Furthermore, we take the mass of the Higgs boson ( $m_H$ ) to be 125 GeV, and use the CT14 NNLO PDFs [25] in this study, with the renormalization scale set at  $\tilde{\mu} = m_H$ .

In Fig. 2, we compare various differential cross sections of the Higgs boson production associated with one inclusive jet at the LHC. The result of the resummation calculations (resum), with two different  $\hat{\mu}$  scales ( $Q$  and  $P_{J\perp}$ ), are denoted by the solid and dotted lines, respectively. The LO result from MCFM, which is the production of Higgs boson plus one inclusive jet with non-zero  $q_\perp$ , is denoted by the dash-dotted lines. The NLO result from MCFM, which is the production of Higgs boson plus two separate jets up to one-loop in QCD, is denoted by the dashed lines.

In the total transverse momentum  $q_\perp$  distribution plot, we find that fixed order calculations (MCFM at LO or NLO) cannot describe the small  $q_\perp$  region. The resummation calculation with the resummation scale ( $\hat{\mu}$ ) chosen to be the jet transverse momentum ( $P_{J\perp}$ )

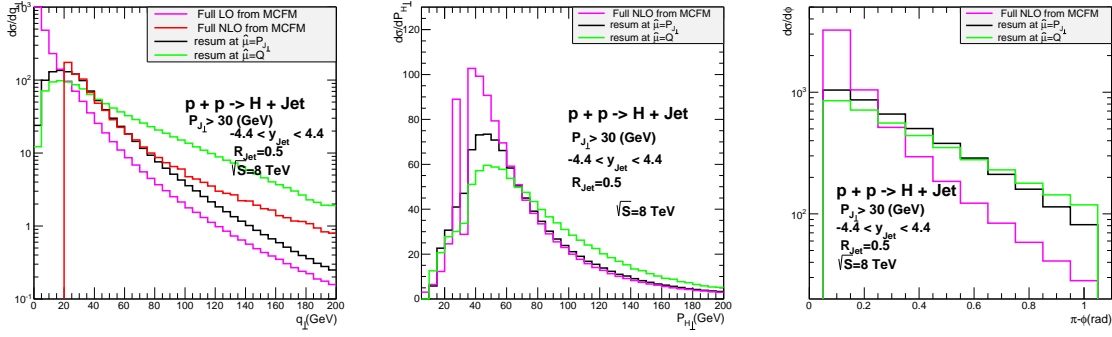


FIG. 2: The differential cross sections of Higgs boson plus one jet production at the LHC as functions of the total transverse momentum  $q_{\perp}$ , the Higgs boson transverse momentum  $P_{H\perp}$ , and the azimuthal angle  $\phi$  between Higgs boson and the leading jet. Here, we compare the resummation predictions (resum), with resummation scale set to be  $P_{J\perp}$  (solid line) and  $Q$  (dotted line), respectively, to the LO result from MCFM (dash-dotted line) with non-zero  $q_{\perp}$ , and the NLO result from MCFM (dashed line) which is the production rate of Higgs boson plus two separate jets up to one-loop in QCD.

predicts a well behaved  $q_{\perp}$  distribution in the small  $q_{\perp}$  region because large logs there have been properly resummed, and its prediction also nicely merges with the full NLO MCFM result as  $q_{\perp}$  approaches to about  $m_H/2$ . On the other hand, the resummation calculation with the resummation scale chosen to be the invariant mass ( $Q$ ) of Higgs boson and jet predicts a too large rate in the large  $q_{\perp}$  region. In the Higgs boson transverse momentum  $P_{H\perp}$  distribution plot, we find that the full NLO MCFM prediction cannot describe  $P_{H\perp}$  near the threshold region, which is the so-called Sudakov-shoulder singularity problem in fixed-order calculations. In contrast, the resummation predictions can be directly compared to the upcoming precision experimental data at the LHC. Again, it shows that the resummation calculation with  $\hat{\mu}$  chosen to be  $P_{J\perp}$  cannot only describe the threshold region, but also agrees well with the NLO MCFM prediction in the large  $P_{J\perp}$  region. For completeness, we also show the comparison of the azimuthal angle  $\phi$  between Higgs boson and the leading jet, in which the resummation calculations differ from the fixed-order prediction, after resumming the effect from multiple gluon radiations in both initial and final state.

In summary, we have applied the TMD resummation theorem to study the production of the Higgs boson associated with one inclusive jet at the LHC. We show that the proper renormalization scale to be used in the resummation calculation is the transverse momentum of the leading jet, and only the resummation calculation can reliably predict various differential cross sections, to be tested by the upcoming precision data at the LHC.

We thank Jianwei Qiu, Bo-Wen Xiao for discussions and comments. We also thank Xiao Feng Luo for helpful discussion. This work is partially supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under contract number DE-AC02-05CH11231, and by the U.S. National Science Foundation under Grant No. PHY-1417326.

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